Computational Learning in Dynamic Logics
Extra Exercises
Day 1

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@NASSLLI, June 2025

Ex. 1. Give an informal scenario that could be described by the following formula:

$$K_1 K_2 p \ \wedge \neg K_2 K_1 K_2 p$$

Ex. 2. Write the following sentence in epistemic logic:
Trump doesn't know whether Al-Assad knows that Obama knows that Al-Assad knows that there was a chemical-weapon attack in Syria.

Ex. 3. Consider the following scenario.

Alice, Bob and Carol each draw a card from a stack of three cards. The cards are J♠, Q♠ and A♠ with backsides indistinguishable. Players can only see their own card, but not the cards of other players. They do see that other players only hold a single card, and that this cannot be their own card, and they do know that all the players know that, etc.

(a) Let JQA stand for the deal: Alice holds J, Bob holds Q, and Carol holds A, etc, and propositions like q_a stand for "Alice holds the queen". Represent the epistemic situation as a possible world model.

(b) Answer the following questions.

1. In JAQ, what does Alice know about Carol's card?
2. And is it the case here that $K_a(j_c \vee j_b)$?
3. Describe in epistemic logic what Alice knows in situation JAQ.
4. In the world JQA, does Alice consider it possible that Alice has Q?
5. In AJQ, is it the case that Bob knows that Carol considers it possible that he has J?

Ex. 4. Consider the following situations.
A. A coin gets tossed under the cup. It lands heads up. Alice and Bob are present, but neither of them can see the outcome, and they both know this.

B. Now Bob leaves the room for an instant. After he comes back, the cup is still over the coin. But Bob realises that Alice might have taken a look (in fact she didn't look). Alice also realizes that Bob considers this possible.

Represent the situations A and B as epistemic models.

Ex. 5. Three children are playing outside together, lets call them a, b and c. Now it happens that during their play all of them get mud on their foreheads. Each can see mud on others but not on his own forehead. Along comes the father, who says, "At least one of you have mud on your forehead". The father then asks the following question, over and over: "Does any of you know whether you have mud on your own forehead?" Assuming that all the children are perceptive, intelligent, truthful, and they answer simultaneously, what will happen? Surprisingly, after the father asks the question for the third time all muddy children will say "yes".

(a) Draw the Pointed Epistemic Model of the initial situation (before the Father's announcements). Remember to designate the actual world, let us refer to is as w.

(b) For each of the following statements decide if they are true or false.

1. $(M, w) \models K_a \neg m_b$
2. $(M, w) \models K_a m_c$
3. $(M, w) \models \neg K_a m_a$
4. $M \models C(m_b \rightarrow K_a m_b)$
5. $M \models C(\neg m_b \rightarrow K_a \neg m_b)$
6. $(M, w) \models E(m_a \vee m_b \vee m_c)$
7. $(M, w) \models E^2(m_a \vee m_b \vee m_c)$

(c) Formalise Father's announcement in propositional logic. How will the model change when children hear the public announcement? How will it change subsequently, until the children arrive at knowing their state?