Computational Learning in Dynamic Logics

In-class Practice, Day 1

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Practice: Epistemic Logic

Exercise 1. The point of this exercise is to get you familiar with the language (syntax) of Epistemic Logic, and get a feel for what sorts of things we can use it to express. As a reminder, formulas in Epistemic Logic have the following syntax:

$$\varphi := \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi$$

Work together with your group to complete the following table.

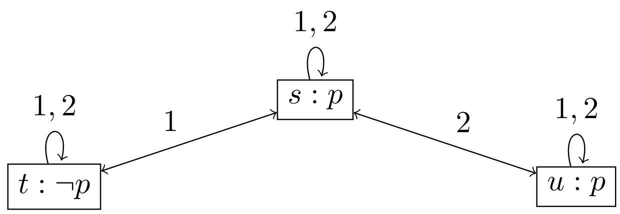
Expression in English	**Epistemic Logic Formula**
I know that φ	$K_i\varphi$
I don't know that φ	$\neg K_i\varphi$
I know that not φ	$K_i\neg\varphi$
I know that I know φ	
I know φ, but I don't know *that*	
I consider φ possible	
I don't know whether φ	

(when you are finished, flip to back of the page for Exercise 2)

Exercise 2. Now let's get our hands dirty with the semantics of Epistemic Logic. As a reminder, the truth of a formula φ at a state s is given by:

$$\begin{array}{lll} (M,s) \models \top & & \text{always} \\ (M,s) \models p & \text{iff} & s \in v(p) \\ (M,s) \models \neg\varphi & \text{iff} & \text{not } (M,s) \models \varphi \\ (M,s) \models \varphi \wedge \psi & \text{iff} & (M,s) \models \varphi \text{ and } (M,s) \models \psi \\ (M,s) \models K_i\varphi & \text{iff} & \text{for all } v \text{ with } (s,v) \in \mathcal{K}_i, (M,v) \models \varphi \end{array}$$

Here is an example of an Epistemic model:



In it, p is the proposition p: "It is sunny in Copenhagen", and we have two agents, 1 and 2. For sake of this exercise, say the real world is state s (it is actually sunny in Copenhagen). Observe that agent 1 (say, Caleb) considers it possible that it is not sunny in Copenhagen, but agent 2 (say, Nina) must face facts because she looks out the window in Copenhagen and sees that it is Sunny.

Using the semantics above, work together with your group to determine the truth of the following formulas (circle your answer).

Expression	**Is it the case?**		**Explanation**
$(M,s) \models p$	**yes**	no	p is true at state s.
$(M,s) \models K_2 p$	**yes**	no	p is true in every state from s that 2 considers possible.
$(M,s) \models K_1 p$	yes	no	
$(M,s) \models \neg K_1 p$	yes	no	
$(M,s) \models K_1(K_2 p \vee K_2 \neg p)$	yes	no	
$(M,s) \models \neg K_2 \neg K_1 p$	yes	no	