Computational Learning in Dynamic Logics
Extra Exercises
Day 2

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Ex. 1. Comparing $[!\varphi]$ and $\langle !\varphi \rangle$. Let us define the semantic of $\langle !\varphi \rangle$ in the following way:

$$(M, s) \models \langle !\varphi \rangle \psi \text{ iff } (M, s) \models \neg [!\varphi] \neg \psi.$$

a. Decide if $(M, s) \models \langle !\varphi \rangle \psi$ is equivalent to:

$$(M, s) \models \varphi \text{ or } (M|\varphi, s) \models \psi.$$

b. Justify your answer.

c. What is the intuitive meaning of $\langle !\varphi \rangle$?

Why is it not '$!(\varphi \wedge \psi)$'? In other words, find a counterexample to:

$$\models [!\varphi][!\psi]\theta \leftrightarrow [!(\varphi \wedge \psi)]\theta.$$

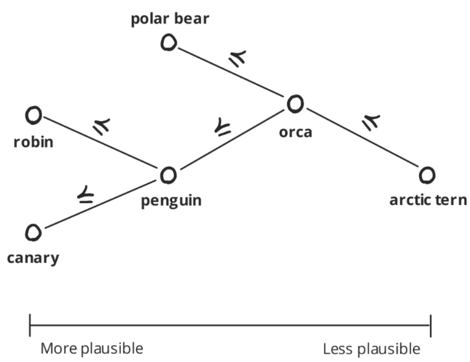
Ex. 2. In Lecture 2, the following validity expressed how to perform sequential composition of announcements:

$$\models [!\varphi][!\psi]\theta \leftrightarrow [!(\varphi \wedge [!\varphi]\psi)]\theta.$$

Why not '$!(\varphi \wedge \psi)$'? In other words, find a counterexample to:

$$\models [!\varphi][!\psi]\theta \leftrightarrow [!(\varphi \wedge \psi)]\theta.$$

Ex. 3. Consider the plausibility model M over animals that we discussed in class:



Let the set of propositions be $Prop = \{\textsc{bird}, \textsc{polar}, \textsc{mammal}, \textsc{fly}, \textsc{penguin}, \textsc{robin}\}$, with the valuations:

$[\![\textsc{bird}]\!] = \{\text{robin}, \text{canary}, \text{penguin}, \text{arctic tern}\}$ $\qquad$ $[\![\textsc{penguin}]\!] = \{\text{penguin}\}$
$[\![\textsc{polar}]\!] = \{\text{polar bear}, \text{penguin}, \text{orca}, \text{arctic tern}\}$ $\qquad$ $[\![\textsc{robin}]\!] = \{\text{robin}\}$
$[\![\textsc{mammal}]\!] = \{\text{polar bear}, \text{orca}\}$
$[\![\textsc{fly}]\!] = \{\text{robin}, \text{canary}, \text{arctic tern}\}$

Think of ROBIN as the classification or predicate "is a robin," and 'robin' as an individual robin.

Determine the validity of the following formulas. (Recall that $M \models \varphi$ whenever $M, s \models \varphi$ for all states s in the model. Notice: the semantics of $\mathbf{B}^{\varphi}\psi$ do not depend on the state s.)

Expression	**Does it hold?**	**Explanation**
$M \models \textsc{bird} \rightarrow \textsc{fly}$	yes no	
$M \models \textsc{penguin} \rightarrow \textsc{fly}$	yes no	
$M \models \mathbf{B}^{\textsc{bird}}(\textsc{fly})$	yes no	
$M \models \mathbf{B}^{\textsc{polar}}(\textsc{fly})$	yes no	
$M \models \mathbf{B}^{\textsc{bird} \wedge \textsc{polar}}(\neg\textsc{fly})$	yes no	
$M \models \mathbf{B}^{\textsc{bird} \wedge \neg\textsc{polar}}(\textsc{fly})$	yes no	

Ex. 4. Assume Bob's believes set p, $p \leftrightarrow q$, and $\neg r$. Come up with an appropriate prior plausibility order on W (the set of all possible truth assignments over p, q, r), which satisfies both of the requirements below:

1. after minimal revision with r Bob would believe that q;
2. after lexicographic revision with $p \rightarrow q$ Bob would believe that $\neg p$.