Computational Learning in Dynamic Logics

Extra Exercises, Day 2

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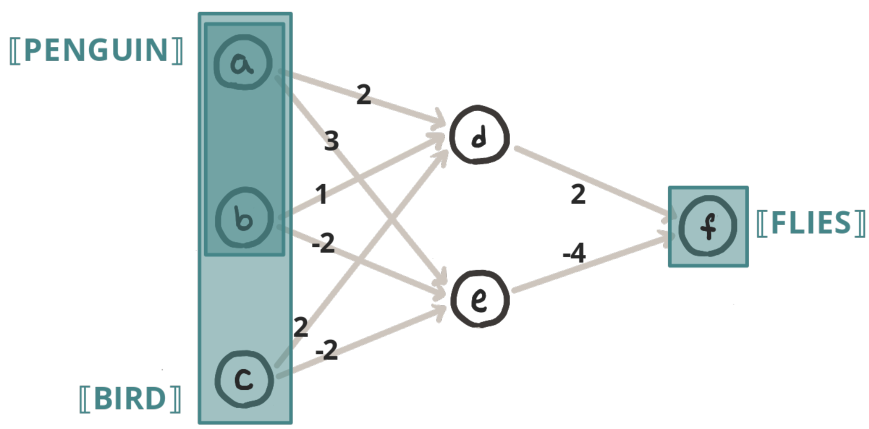
Ex 1. Consider the following binary neural network model $\mathcal{N} = \langle N, E, W, A, \eta, V\rangle$, where

- N, E, W are as shown
- $A: \mathbb{Q} \to \{0,1\}$ is given by $A(x) = 1$ iff $x > 0$
- $\eta = 1$
- Prop. valuations are given by $[\![\text{BIRD}]\!] = \{a,b,c\}$, $[\![\text{PENGUIN}]\!] = \{a,b\}$, $[\![\text{FLIES}]\!] = \{f\}$



(a). Calculate $\mathsf{Clos}([\![\text{BIRD}]\!])$ and $\mathsf{Clos}([\![\text{PENGUIN}]\!])$. Do they each contain $[\![\text{FLIES}]\!]$?

(b). Now suppose our agent observes a Puffin, an animal very similar to a penguin that *does* fly. Let $[\![\text{PUFFIN}]\!] = \{b,c\}$. First, calculate $\mathsf{Clos}([\![\text{PUFFIN}]\!])$. Then explain what happens when we apply $\mathsf{Hebb}(\mathcal{N}, [\![\text{PUFFIN}]\!])$ repeatedly 3 times.

(c). Evaluate the truth of the following formulas.
Hint: $\mathbf{C}\varphi \to \psi$ is logically equivalent to its dual $\psi \to \langle \mathbf{C}\rangle\varphi$. You can replace it with this latter form, which is easier to calculate the semantics for.

Expression	Does it hold?	
$\mathcal{N} \models \mathbf{A}(\text{PENGUIN} \to \text{BIRD})$	yes	no
$\mathcal{N} \models \mathbf{A}(\text{PUFFIN} \to \text{BIRD})$	yes	no
$\mathcal{N} \models \mathbf{A}(\mathbf{C}(\text{PENGUIN}) \to \text{FLIES})$	yes	no
$\mathcal{N} \models [\text{PUFFIN}]_{\mathsf{Hebb}}(\mathbf{A}(\mathbf{C}(\text{PENGUIN}) \to \text{FLIES}))$	yes	no
$\mathcal{N} \models [\text{PUFFIN}]_{\mathsf{Hebb}}[\text{PUFFIN}]_{\mathsf{Hebb}}[\text{PUFFIN}]_{\mathsf{Hebb}}(\mathbf{A}(\mathbf{C}(\text{PENGUIN}) \to \text{FLIES}))$	yes	no

Ex 2. Come up with a binary neural network model (complete with weights and an activation function) that contains a cycle, yet every activation state S stabilizes to a unique state $\mathsf{Clos}(S)$.

Ex 3. Recall that in Epistemic Logic, an agent knows all the logical consequences of her knowledge, i.e.,

$$\models (K\varphi \wedge K(\varphi \rightarrow \psi)) \rightarrow K\psi$$

Note that this is logically equivalent to the following dual formula. (It takes some work to show, and it's a good modal logic side-exercise.)

$$\models \langle K \rangle \psi \rightarrow (\langle K \rangle \varphi \vee \langle K \rangle (\neg\varphi \wedge \psi))$$

Show that the instance of this axiom for $\langle \mathbf{C} \rangle$ is *not* valid for neural network models, i.e.,

$$\not\models \langle \mathbf{C} \rangle \psi \rightarrow (\langle \mathbf{C} \rangle \varphi \vee \langle \mathbf{C} \rangle (\neg\varphi \wedge \psi))$$

Hint: Consider φ, ψ as propositions p, q. Come up with a binary neural network model $\mathcal{N}$, a neuron $w \in N$, and valuations for p, q that serve as a counterexample. You should be able to do it using only three nodes.

Ex 4. Explain why the following formula is valid for single-step $[\varphi]_{\mathsf{Hebb}}$ over binary neural network models:

$$\models [\langle \mathbf{C} \rangle \varphi]_{\mathsf{Hebb}} \psi \leftrightarrow [\varphi]_{\mathsf{Hebb}} \psi$$

Hint: A high-level explanation is fine; the proof of this is a bit more complicated, since we have to deal with potentially recurrent edges. See the lecture notes for the full proof.